Charge and Statistics of Quasiholes in Pfaffian States of Composite Fermion Excitations

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Abstract

The charge of quasiparticles in Pfaffian states of composite fermion excitations (the presence of which is indicated by recent experiments) is found. At the filling fraction of the Pfaffian state $\nu = p/q$ (of the lowest Landau level) the charge is $\pm e/(2q)$. As in the case of the Pfaffian state of electrons the statistics of N_{qh} quasiholes in the Pfaffian state corresponds to the spinor representation of $U(1) \times SO(2N_{qh})$ (the continuous extension of the braid group). Here $U(1)$ is given by the phase factor $e^{i(\frac{1}{8}+\frac{1}{4m})\pi}$ with $m = 1 + \alpha$, α – the exclusion statistics parameter of Jain quasiparticles. The possiblity of Read-Rezayi states of Jain quasiparticles is also discussed.

Recently, the non-Laughlin and non-Jain states were observed in the fractional quantum Hall effect [1]. They can be interpreted as condensed states of composite fermion excitations [2,3]. Even-denominator states (i. e. at filling fractions with an even denominator) can be seen as Pfaffian states of Jain quasiparticles. Odd-denominator states correspond to the condensed states of composite fermion excitations which can either be Laughlin [4] (or Jain) states or condensed states of different origin. One possiblity (as competing to Laughlin-Jain states) is the class of states proposed by Read and Rezayi [5] for electron states in higher Landau levels (Read-Rezayi states result from clustering of a number of particles, the Pfaffian state is the state of pairs of particles [6]). The other proposed states for the origin of odd-denominator states are Halperin paired states [7–9].

The composite fermion approach predicts the so-called Jain states at filling fractions of the form $\nu = \frac{n_0}{2p_0 n_0 + \beta_0}$ (where $2p_0$ – the Chern-Simons composite fermion parameter being an even number, n_0 – the number of effective shells filled, β_0 – the sign of the effective field with respect to the external magnetic field). The effective field is found when the mean Chern-Simons composite fermion field is added to the external magnetic field. Let us introduce the condensed states of composite fermion excitations (following [2]) which appear at the filling fraction of the form:

$$\nu_e^{-1} = 2p_0 + \frac{\beta_0}{n_0 + \nu_1} \tag{1}$$

where ν_1 is the fraction in which the (n_0+1)-th effective shell is partially filled. All recently observed states [1] can be described within (1) (e. g. $\nu_e = 4/11$, with $p_0 = 1$, $n_0 = 1$, $\beta_0 = 1$, $\nu_1 = 1/3$). The states 4/13, 5/17, correspond to the $\beta_0 = -1$ case (the effective field is opposite to the external magnetic field). The state 7/11 can either be seen as the $\beta_0 = -1$ state (with $n_0 = 2$) or as the 4/11 state of holes conjugated to electrons. The partial fillings of interest (as observed in the experiment) are $\nu_1 = 1/3$, 2/3, and 1/5. Even-denominator states correspond to $\nu_1 = 1/2$. In this paper we are also going to refer to Read-Rezayi states defined at filling fractions (in the fermion representation of fractional statistics particles):

$$\nu_1 = \frac{k}{2+k} \tag{2}$$

($k = 2$ for the Pfaffian, for $k = 1$ the Laughlin 1/3 state is found). All observed hierarchical states would correspond to even-k Read-Rezayi states, i. e. $\nu_1 = 1/2$ (the Pfaffian $k = 2$), $\nu_1 = 2/3$ (or by the particle-hole conjugation $\nu_1 = 1/3$) – $k = 4$, $\nu_1 = 4/5$ (by the particle-hole conjugation $\nu_1 = 1/5$) – $k = 8$ (which correponds to the $\nu = 6/17$ state). In the last case it seems that the Laughlin 1/5 state should rather be preferred over the Read-Rezayi clusters of 8 quasiparticles. Assuming the possibility of Read-Rezayi states the question arises why only these three values of k would be of interest. If one would have the $k = 4$ Read-Rezayi state, why a $k = 3$ state would not appear. Even worse, why the $k = 8$ state if there would be no $k = 5, 6, 7$ states. (The case of $k = 6$ would correspond to the $\nu_1 = 1/4$ Pfaffian state via particle-hole conjugation, if observed). The simple test for Read-Rezayi states would be for example the observation of the $\nu_1 = 3/7$ state of composite fermion excitations (it is the first Jain state which can not be represented as a Read-Rezayi state). However, we should stress that the Pfaffian state of composite fermion excitations is seen as the best candidate for recently discovered even-denominator hierarchical states 3/8 and 3/10.

It is worth to notice that the observed 3/8 state corresponds to the half-filling of Laughlin quasielectron shell (Laughlin quasielectrons of the 1/3 state). However, the 3/10 state corresponds to the 1/4 filling of the Laughlin quasihole shell. It is perhaps better to see the 3/10 state as the half-filled quasielectron shell, but seen within the opposite effective field picture (or, equivalently, to the half-filled quasihole level of quasiholes of the 2/7 state). Within the composite fermion picture then, both states (3/8 and 3/10) correspond to a half-filling of the first excited effective Landau level. It is known that the Pfaffian state of electrons is proposed for the $\nu = 5/2$ quantum Hall state (electrons in the first excited Landau level, two underlying levels are occupied by electrons of opposite spins). Also, the Read-Rezayi states are proposed for electrons in the first excited (or higher) Landau level. Hence, it is interesting to consider also Read-Rezayi states of quasiparticles as natural candidates (competing with the picture of Laughlin or Jain states of quasiparticles). Maybe, except for the $\nu = 6/17$ state (the 1/5 state of quasielectrons, or the 4/5 state of conjugated

holes – the Read-Rezayi state for $k = 8$ – 8 holes make a cluster – as discussed above).

At present in the case of odd-denominator states the first guess aims at Laughlin or Jain states of quasiparticles [2,3]. One possible competing mechanism of incompressibility may be due to clustering of quasiparticles as discussed by Read and Rezayi (in the case of electrons) [5]. Laughlin (or Jain) states of Jain quasiparticles are not seen in numerical studies (e. g. in spherical systems [10]) but the studies are far from being conclusive (the number of quasiparticles is very low when exact diagonalization of electrons is considered, usually 3-4 quasiparticles [2,10]). The approximate numerical methods for higher numbers of quasiparticles are also proposed [2,11,12,9] but their accuracy seems to be difficult to estimate.

There are examples in numerical studies (on a sphere) which support the Pfaffian states of composite fermion excitations (which should correspond to the 3/8 states), e. g. the states with the $L = 0$ ground state: $N_e = 14$ $2S = 33$ ($2S_{qe} = 9$, $N_{qe} = 6$) [11], $N_e = 12$ $2S = 29$ ($2S_{qe} = 9$, $N_{qe} = 4$) [2,12] (N_e electrons are in the angular momentum shell S, N_{qe} quasielectrons are in the effective shell S_{qe}). The former can be seen as the Pfaffian state of quasielectrons ($2S_{qe} = 2N_{qe} - 3$) [13], the latter is related by the particle-hole conjugation (in the quasielectron shell). There are also approximate results for a higher number of quasiparticles pointing at a Pfaffian state (e. g. for $N_{qe} = 10$, $2S_{qe} = 17$ in [9]).

We are going to define the charge of quasiparticles in Pfaffian (and Read-Rezayi) states of composite fermion excitations. Note that as Jain quasiparticles are anyons they can be seen in the fermion (or boson) representation (with the appropriate Chern-Simons field included). We use the strong-pairing (strong-clustering) limit which is valid if charge of excitations is considered [13,5,8]. The (non-Abelian) statistics of quasiholes in Pfaffian states corresponds to the spinor representation of $U(1) \times SO(2N_{qh})$ (N_{qh} – a number of quasiholes) of the dimension $2^{N_{qh}-1}$. In the present case one can consider Pfaffian states of Jain quasiparticles in the boson representation. It means we first consider the Pfaffian state of bosons, and next we introduce the Chern-Simons field appropriate for the fractional statistics. Such operation affects only the $U(1)$ term of the representation (Fradkin *et al.* [14] discussed the similar

case where the Pfaffian fermion state is found within the boson representation of fermions). The Pfaffian state (in the plane, $z_i = x_i + iy_i$) is defined as:

$$\Psi_{Pf} = Pf(\frac{1}{z_i - z_j})\prod_{i>j}(z_i - z_j)^m e^{-\frac{\sum |z_i|^2}{4a_0^2}} \quad , \tag{3}$$

a_0 is the magnetic length, m is odd for bosons, and is even for fermions (at the respective filling fraction $1/m$). One can notice that m appears only in the Laughlin-like part of the wave function (3). For four quasiholes in the Pfaffian state the braiding matrix is [14]

$$\frac{e^{i\pi(\frac{1}{8}+\frac{1}{4m})}}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \tag{4}$$

(in the spinor representation). We note that the second term in the phase factor $(\frac{\pi}{4m})$ can be found in the strong-pairing limit [13,15,16]. Hence, the value of quasihole charge is $\frac{1}{4m}Q_c$ where Q_c is the charge of pairs of particles, for $Q_c = 2e$ one gets the charge $\frac{e}{2m}$. The Pfaffian state of anyons (Jain quasiparticles) corresponds to a noninteger m [13] (which describes the statistics), and we define $m = 1 + \alpha$ where α is the exclusion statistics parameter of anyons [17] (in the magnetic field, at the effective filling 1/2, in the fermion representation, $\alpha \geq 0$).

We discuss below the strong-pairing (strong-clustering) limit. Let us consider the filling fraction of the form

$$\nu_e^{-1} = 2p_0 + \frac{\beta_0}{1+\nu_1} \tag{5}$$

ν_1 is the filling fraction of composite fermion excitations in the effective $n_0 = 2$ shell. We assume Read-Rezayi states of quasielectrons (for the Pfaffian $k = 2$), hence [5]

$$\nu_1^{-1} = 1 + \frac{2}{k} \ . \tag{6}$$

The total flux entering the system is

$$\Phi^{ex} = N_e(\nu_e^{-1})\frac{hc}{e} \ . \tag{7}$$

One gets $N_e = \frac{1+\nu_1}{\nu_1}N_{qe}$, $Q_{qe} = e/q_p$ (quasielectrons partially fill the first excited effective shell), q_p is the denominator of the parent state [18]. Since k quasielectrons make a cluster one gets $N_c = \frac{N_{qe}}{k}$ (the number of clusters), $Q_c = kQ_{qe}$, and

$$\Phi^{ex} = \nu_e^{-1}\frac{1+\nu_1}{\nu_1}k^2\frac{1}{q_p}N_c\frac{hc}{Q_c} \ . \qquad (8)$$

In the case of N_c anyons in the Laughlin state [17] (of charge Q_c and the statistics $\theta/\pi = (\alpha_{str}^{qh})^{-1}(mod\, 2)$)

$$\Phi^{ex} = (\alpha_{str}^{qh})^{-1}\frac{hc}{Q_c}N_c \qquad (9)$$

with quasihole excitations of the exclusion statistics α_{str}^{qh}, and the quasihole charge $Q = \alpha_{str}^{qh}Q_c$. We assume that k-clusters condense at a Laughlin state, then the excitations charge Q is (if absolute values of charges are considered)

$$Q^{-1} = \nu_e^{-1}\frac{1+\nu_1}{\nu_1}k^2\frac{1}{q_p}\frac{1}{Q_c} \qquad (10)$$

$$Q^{-1}e = \nu_e^{-1}\frac{1+\nu_1}{\nu_1}k \ . \qquad (11)$$

One gets then

$$eQ^{-1} = 4p_0 + 2\beta_0 + k(4p_0 + \beta_0) \qquad (12)$$

and

$$\nu_e = \frac{2+2k}{eQ^{-1}} \ . \qquad (13)$$

For k even the filling fraction can be simplified (the numerator and the denominator in ν_e may be divided by two). Hence, the charge of excitations equals $\pm e/q$ for k odd and $\pm e/(2q)$ for k even. The same conclusion can be reached at the filling fraction of the form:

$$\nu_e^{-1} = 2p_0 + \frac{\beta_0}{2-\nu_1} \qquad (14)$$

where holes conjugated to quasielectrons are considered (Jain quasiholes of the state $\nu_e^{-1} = 2p_0 + \frac{\beta_0}{2}$). In that case

$$eQ^{-1} = k(2p_0 + \beta_0) + 8p_0 + 2\beta_0 \qquad (15)$$

and

$$\nu_e = \frac{4+k}{eQ^{-1}} \ . \tag{16}$$

The charge of quasiparticles in fractional quantum Hall systems can be detected, e. g. in "shot-noise" experiments [19–22].

The statistics of excitations in the strong-pairing limit can also be determined. Let us consider the Pfaffian, i. e. $k = 2$, then

$$eQ^{-1} = 4(3p_0 + \beta_0) = [\alpha^{qh}_{str} Q_c]^{-1} e \ , \tag{17}$$

α^{qh}_{str} is the exclusion statistics parameter of quasiholes in the strong-pairing limit. When one considers the statistics of quasiholes within the strong-pairing limit one finds

$$\alpha^{qh}_{str} = \frac{1}{Q_c Q^{-1}} = \frac{q_p}{2eQ^{-1}} \ , \tag{18}$$

$q_p = 2p_0 n_0 + \beta_0$ – the denominator of the parent Jain state (here $n_0 = 1$). And

$$\alpha^{qh}_{str} = \frac{\beta_0 + 2p_0}{2 \cdot 4(3p_0 + \beta_0)} \ . \tag{19}$$

Let us assume that $\alpha_{qe} = m - 1$, then $m = \alpha_{qe} + 1 = \frac{2(\beta_0 + 3p_0)}{\beta_0 + 2p_0}$ [17]. Then

$$\alpha^{qh}_{str} = \frac{1}{4m} \ . \tag{20}$$

The same relation can be found for Jain quasiholes in (14). The result (20) is the one which appears in the expression for the statistics of excitations in the Pfaffian state (4). The state 3/8 corresponds to the Pfaffian state of $\alpha_{qe} = 5/3$ quasielectrons [17]. Then the phase factor in (4) is

$$\frac{\pi}{8} + \frac{3}{32}\pi \ . \tag{21}$$

When one considers the Pfaffian state of the conjugated quasiholes (of the 2/5 state, $\alpha_{qh} = 3/5$) one gets $\frac{\pi}{8} + \frac{5}{32}\pi$. Note, however, that both cases correspond to opposite change in the external magnetic field (e. g. quasihole-like, quasielectron-like excitations). At $\nu = 3/10$ we have $\alpha_{qe} = 7/3$ [17], so that the phase factor is $\pi/8 + (3/40)\pi$ (or if one starts from $\nu = 2/7$, $\alpha_{qh} = 3/7$, then one gets the phase $\pi/8 + (7/40)\pi$).

In conclusion we found the charge of quasiparticles in condensed states of composite fermion excitations seen as the Pfaffian and Read-Rezayi states. In the standard hierarchy (e. g. the Haldane hierarchy [18] or the hierarchy of composite fermion excitations [2]) the charge is always $\pm e/q$ where q is the denominator of the filling fraction ($\nu = p/q$; p, q are coprime). However, for Pfaffian or Read-Rezayi states (where k particles make a cluster, $k = 2$ for the Pfaffian) one gets $\pm e/q$ for k odd and $\pm e/(2q)$ for k even (e. g. for the Pfaffian state). Then, the charge of excitations in the proposed Pfaffian states 3/8 and 3/10 should be $\pm e/16$ and $\pm e/20$, respectively. This provides also the possible identification of Read-Rezayi states of Jain quasiparticles (for even k) in charge-detecting experiments (e. g. in "shot-noise" experiments). For example the charge of excitations at $\nu_e = 5/13$ (and $\nu = 4/11$) should be $\pm e/26$ ($\pm e/22$ for $\nu_e = 4/11$) if one deals with Read-Rezayi states of quasiparticles. Hence, for all observed states it should be possible to identify Pfaffian and k-even Read-Rezayi states by future charge-detecting experiments. The Pfaffian state can be further tested by detecting (if possible) the non-abelian statistics of excitations. The non-Abelian statistics of quasiholes in Pfaffian states of Jain quasiparticles is given as the spinor representation of $U(1) \times SO(2N_{qh})$ (the continuous extension of the braid group) with $U(1)$ given by $e^{i(\frac{\pi}{8}+\frac{\pi}{4m})}$ where $m = 1 + \alpha$, α – the exclusion statistics of Jain quasiparticles (at the effective filling fraction 1/2 in the fermion representation). Let us note that also excitations in Read-Rezayi states of Jain quasiparticles (if observed) would obey non-abelian statistics.

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